

A Generalized Gauge Theory of Gravity

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Abstract

In this paper, we discuss a gravitational theory based on the generalized gauge field. Our Lagrangian is invariant not only under local Lorentz transformation and the ordinary gauge transformation but also under a new gauge transformation. We show that the gauge field associated with this new transformation is a second-rank tensor field and that the Einstein-Hilbert term can be derived from our Lagrangian when the gauge field has a vacuum expectation value. We also show that our model provides a Lagrangian for the scalar-tensor theory.

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§1. Introduction

A considerable number of previous studies have explored the origins of scalar fields. In one of the previous studies, a model that regards a scalar field as a type of gauge field was considered. For example, the number of dimensions considered in the “gauge-Higgs unification”^{1),2)} was greater than four, and the extra components in a higher-dimensional gauge field were identified as the Higgs fields. In such a model, the scalar fields are constrained by the gauge principle, and thus, the predictive power of the model is at least as good as that of the Standard Model.

Sogami³⁾ proposed a covariant derivative of the form

$$\partial_\mu - \frac{1}{2}(1 - \gamma_5)A_{L\mu} - \gamma_\mu\phi,$$

where ϕ is the Higgs field. Note that the fields in this covariant derivative contain γ_5 and γ_μ . The gravitational field is a gauge field whose generators are a product of gamma matrices.⁴⁾ In our previous paper,⁵⁾ we considered the generalized gauge field that has various combinations of gamma matrices as generators and showed that the generalized gauge field includes vector field, spin connections, scalar field, and tensor field. Moreover, we identified the derived scalar as the Higgs field and attempted to extend the Standard Model. However, we were unable to determine the gauge transformation with which the gauge field of the scalar field or tensor field was associated. Moreover, our model was invariant under local Lorentz transformation; however, no discussion in terms of gravitational theory was provided in the paper.

In this paper, we further generalize the gauge field discussed in the previous paper and attempt to apply it to gravitational theory in four and five dimensions. The results show that our model has a new gauge invariance, and the gauge field associated with the transformation is a second-rank tensor field. We show that if the tensor field has a vacuum expectation value, the Einstein-Hilbert term can be derived from our Lagrangian. We also show that our model can be used to explain the “scalar-tensor theory.”^{6),7)}

§2. The four-dimensional model

We consider the following Lagrangian

$$\mathcal{L} = \sqrt{-g} \left\{ -\bar{\psi} i \gamma^\mu \mathcal{D}_\mu \psi - \frac{1}{4} \text{Tr}(\mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu}) \right\} \quad (2.1)$$

with

$$\mathcal{D}_\mu \equiv \partial_\mu - ie\mathcal{A}_\mu, \quad \mathcal{F}_{\mu\nu} \equiv \frac{i}{e}[\mathcal{D}_\mu, \mathcal{D}_\nu], \quad (2.2)$$

where Tr is the trace of a 4×4 matrix and \mathcal{A}_μ is any 4×4 matrix that satisfies

$$\gamma_0 \mathcal{A}_\mu^\dagger \gamma_0 = -\mathcal{A}_\mu. \quad (2.3)$$

We adopt the metric signature $(-, +, +, +)$.

We find that (2.1) is invariant under the following transformations:

$$\psi \rightarrow \psi' = U(x)\psi, \quad \gamma_\mu \rightarrow \gamma'_\mu = U(x)\gamma_\mu U^{-1}(x), \quad (2.4)$$

$$\mathcal{A}_\mu \rightarrow \mathcal{A}'_\mu = U(x)\mathcal{A}_\mu U^{-1}(x) + \frac{1}{ie}(\partial_\mu U(x))U^{-1}(x), \quad (2.5)$$

where $U(x)$ is an arbitrary matrix that satisfies

$$\gamma_0 U^\dagger(x) \gamma_0 = -U^{-1}(x). \quad (2.6)$$

Note that γ'_μ satisfies the relations, $\{\gamma'_\mu, \gamma'_\nu\} = 2g_{\mu\nu}$, when $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$.

The transformation $U(x)$ involves gauge transformation,

$$U(x) = e^{iI\varepsilon(x)/2}, \quad (2.7)$$

and local Lorentz transformation,

$$U(x) = e^{\gamma^{ij}\varepsilon_{ij}(x)/16} \quad \text{with } \gamma^{ij} \equiv \frac{\gamma^i \gamma^j - \gamma^j \gamma^i}{2}, \quad (2.8)$$

where I denotes the 4×4 unit matrix. In addition, $U(x)$ involves the following new transformation:

$$U(x) = e^{i\gamma^i \varepsilon_i(x)/2}. \quad (2.9)$$

In general, $U(x)$ involves local transformations with the generators:

$$I, \quad \gamma^i, \quad \gamma^{ij}, \quad \gamma_5 \gamma^i, \quad \gamma_5. \quad (2.10)$$

Here, we have constructed a flat tangent space at every point on the four-dimensional manifold, and we indicate the vectors in the five-dimensional tangent space by subscript Roman letters i, j, k , etc. Greek letters like μ and ν are used to label four-dimensional space-time vectors. Note that γ_i is a constant matrix. Now, we define the *vierbein* (tetrad for four-legged) that is used for the transformation from the x -space to the tangent space, and vice versa, as

$$\begin{aligned} b^i{}_\mu b_{i\nu} &= g_{\mu\nu}, & b^{i\mu} &= g^{\mu\nu} b^i{}_\nu, \\ b^i{}_\mu b_j{}^\mu &= \delta_j^i. \end{aligned} \quad (2.11)$$

We can then define a set of gamma matrices over either the tangent space or the x -space as

$$\gamma^i b_i{}^\mu = \gamma^\mu, \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \quad (2.12)$$

Because (2.1) is invariant under gauge transformation and local Lorentz transformation, \mathcal{A}_μ must involve the gauge field and the spin connection. It is known that an arbitrary 4×4 matrix M can be decomposed into 16 gamma matrices, that is, I , γ^i , γ^{ij} , $\gamma_5 \gamma^i$, and γ_5 , as

$$M = \frac{1}{4} [\text{Tr}(M)I + \text{Tr}(\gamma_5 M)\gamma_5 + \text{Tr}(\gamma_i M)\gamma^i - \text{Tr}(\gamma_5 \gamma_i M)\gamma_5 \gamma^i] \\ - \frac{1}{8} \text{Tr}(\gamma_{ij} M)\gamma^{ij}. \quad (2.13)$$

By substituting $M = \mathcal{A}_\mu$ in (2.13), we derive

$$\mathcal{A}_\mu = \frac{1}{2} \left(A_\mu I + A_\mu^{(5)} \gamma_5 + A_{i\mu} \gamma^i - A_{i\mu}^{(5)} \gamma_5 \gamma^i + \frac{i}{2} \omega_{ij,\mu} \gamma^{ij} \right). \quad (2.14)$$

Here, we define

$$A_\mu \equiv \frac{1}{2} \text{Tr}(\mathcal{A}_\mu), \quad A_\mu^{(5)} \equiv \frac{1}{2} \text{Tr}(\gamma_5 \mathcal{A}_\mu), \\ A_{i\mu} \equiv \frac{1}{2} \text{Tr}(\gamma_i \mathcal{A}_\mu), \quad A_{i\mu}^{(5)} \equiv \frac{1}{2} \text{Tr}(\gamma_5 \gamma_i \mathcal{A}_\mu), \\ \omega_{ij,\mu} \equiv \frac{i}{2} \text{Tr}(\gamma_{ij} \mathcal{A}_\mu). \quad (2.15)$$

Let us investigate how each field in (2.15) changes under the infinitesimal transformation described in (2.7), (2.8), and (2.9).

For (2.7), we consider the trace of \mathcal{A}'_μ in (2.5) and obtain

$$\delta A_\mu = \frac{1}{e} \partial_\mu \varepsilon. \quad (2.16)$$

This implies that A_μ is the usual gauge field.

For (2.8), we multiply \mathcal{A}'_μ in (2.5) from the left by $\frac{i}{2} \gamma_{ij}$ and consider the trace to obtain

$$\delta \omega_{ij,\mu} = \varepsilon_i{}^k \omega_{kj,\mu} + \varepsilon_j{}^k \omega_{ik,\mu} - \frac{1}{4e} \partial_\mu \varepsilon_{ij}. \quad (2.17)$$

From the abovementioned equation, we can identify $\omega_{ij,\mu}$ as the spin connection.

For (2.9), we have

$$\delta A_{i\mu} = \varepsilon^k \omega_{ik,\mu} + \frac{1}{e} \partial_\mu \varepsilon_i, \quad \delta \omega_{ij,\mu} = \varepsilon_i A_{j\mu} - \varepsilon_j A_{i\mu}, \\ \delta A_{i\mu}^{(5)} = i \varepsilon_i A_\mu^{(5)}, \quad \delta A_\mu^{(5)} = i \varepsilon^k A_{k\mu}^{(5)}. \quad (2.18)$$

In other words, $A_{i\mu}$ is a gauge field that is associated with the new transformation described in (2.9).

All the fields represented by (2.15) couple with a universal coupling constant e . Hence, each field represented by (2.15) is interpreted as a type of gravitational field.

We can reexpress \mathcal{L} also in terms of (2.14). The calculation, however, can be markedly simplified by extending our model to five dimensions. In the next section, we will show the extension of the present model to five dimensions and discuss the features of our model in detail.

§3. The five-dimensional model

We consider the following action

$$S = \int d^5x \sqrt{-g} \left\{ -\bar{\psi} i \gamma^M \mathcal{D}_M \psi - \frac{\chi}{4} \text{Tr}(\mathcal{F}^{MN} \mathcal{F}_{MN}) \right\} \quad (3.1)$$

with

$$\mathcal{D}_M \equiv \partial_M - ie \mathcal{A}_M, \quad \mathcal{F}_{MN} \equiv \frac{i}{e} [\mathcal{D}_M, \mathcal{D}_N], \quad (3.2)$$

where $M = 0, 1, 2, 3, 4$ and \mathcal{A}_M is any 4×4 matrix that satisfies

$$\gamma_0 \mathcal{A}_M^\dagger \gamma_0 = -\mathcal{A}_M, \quad (3.3)$$

and γ^M represents the five-dimensional gamma matrices,

$$\gamma^M = (\gamma^\mu, \gamma^4) \quad \text{with } \gamma^4 \equiv \gamma_5. \quad (3.4)$$

Since \mathcal{A}_M has a mass dimension of 1, as seen from (3.2), we multiply the bosonic part of the Lagrangian by a coefficient χ whose mass dimension is 1, so that S becomes dimensionless.

(3.1) is invariant under the following transformations:

$$\psi \rightarrow \psi' = U\psi, \quad \gamma_M \rightarrow \gamma'_M = U\gamma_M U^{-1}, \quad (3.5)$$

$$\mathcal{A}_M \rightarrow \mathcal{A}'_M = U\mathcal{A}_M U^{-1} + \frac{1}{ie} (\partial_M U) U^{-1}, \quad (3.6)$$

where

$$U = e^{iI\varepsilon(x)/2}, \quad e^{\gamma^{ab}\varepsilon_{ab}(x)/16}, \quad e^{i\gamma^a\varepsilon_a(x)/2}. \quad (3.7)$$

Here, we indicate the vectors in the five-dimensional tangent space by subscript Roman letters a, b, c , etc. Moreover, we use capital letters such as M and N to label the five-dimensional space-time vectors.

Equation (2.13) can be rewritten as

$$M = \frac{1}{4} [\text{Tr}(M)I + \text{Tr}(\gamma_a M)\gamma^a] - \frac{1}{8} \text{Tr}(\gamma_{ab} M)\gamma^{ab}. \quad (3.8)$$

By substituting $M = \mathcal{A}_M$ in (3.8), we obtain

$$\mathcal{A}_M = \frac{1}{2}(A_M I + A_{aM}\gamma^a + \frac{i}{2}\omega_{ab,M}\gamma^{ab}). \quad (3.9)$$

Here, we define

$$A_M \equiv \frac{1}{2}\text{Tr}(\mathcal{A}_M), \quad A_{aM} \equiv \frac{1}{2}\text{Tr}(\gamma_a \mathcal{A}_M), \quad \omega_{ab,M} \equiv \frac{i}{2}\text{Tr}(\gamma_{ab} \mathcal{A}_M). \quad (3.10)$$

Note that $A_\mu^{(5)}$ and $A_{i\mu}^{(5)}$ in the four-dimensional model have been included in A_{aM} and $\omega_{ab,M}$, respectively.

Using (3.9), we can calculate the field strength as

$$\begin{aligned} \mathcal{F}_{MN} &= \frac{i}{e}[\mathcal{D}_M, \mathcal{D}_N] \\ &= \frac{1}{2}(D_M A_{aN} - D_N A_{aM})\gamma^a - \frac{i}{2}e A_{aM} A_{bN}\gamma^{ab} \\ &\quad + \frac{i}{4}R_{ab,MN}\gamma^{ab} + \frac{1}{2}F_{MN}I, \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} D_M A_{aN} &\equiv \partial_M A_{aN} + e\omega_a{}^b{}_{,M} A_{bN}, \\ R^{ab}{}_{,MN} &\equiv \partial_M \omega^{ab}{}_{,N} - \partial_N \omega^{ab}{}_{,M} + e(\omega^a{}_{c,M} \omega^{cb}{}_{,N} - \omega^a{}_{c,N} \omega^{cb}{}_{,M}), \\ F_{MN} &\equiv \partial_M A_N - \partial_N A_M. \end{aligned} \quad (3.12)$$

We know that the Riemann curvature tensor $R^P{}_{Q,MN}$ can be written as

$$R^P{}_{Q,MN} = b_a{}^P b_{bQ} R^{ab}{}_{,MN}. \quad (3.13)$$

Using this field strength, we determine the bosonic part of the Lagrangian:

$$\begin{aligned} &-\frac{1}{4}\text{Tr}(\mathcal{F}^{MN}\mathcal{F}_{MN}) \\ &= \frac{1}{4}e A_a{}^M A_b{}^N R^{ab}{}_{,MN} - \frac{1}{4}(D_M A_{aN} - D_N A_{aM})(D^M A^{aN} - D^N A^{aM}) \\ &\quad - \frac{1}{4}e^2(A_{aM} A^{aM} A_{bN} A^{bN} - A_{aM} A^{aN} A_{bN} A^{bM}) \\ &\quad - \frac{1}{8}R^{CD}{}_{,MN} R_{CD,}{}^{MN} - \frac{1}{4}F^{MN}F_{MN}. \end{aligned} \quad (3.14)$$

We intend to include the Ricci curvature R in (3.14). Now, we propose two methods to obtain R . The first method involves the use of an expectation value. Let us assume that A_{aM} has an expectation value given by

$$\langle A_{aM} \rangle = \frac{1}{2}v(y)b_{aM} \quad (3.15)$$

and write

$$A_{aM} = \frac{1}{2}v(y)b_{aM} + A'_{aM}, \quad (3.16)$$

where $v(y)$ is a function of the fifth coordinate y . The validity of the vacuum expectation value will be discussed in the last section. In general, the covariant derivative of the *funfbein* (tetrad for five-legged) should be zero,

$$\partial_M b_{aN} + e\omega_{ab,M}b^b{}_N - \Gamma^P{}_{NM}b_{aP} = 0, \quad (3.17)$$

where $\Gamma^C{}_{NM}$ is the Christoffel symbol. Using (3.16) and (3.17), we obtain

$$\begin{aligned} & -\bar{\psi}i\gamma^M\mathcal{D}_M\psi \\ &= -\bar{\psi}i\gamma^M\partial_M\psi - \frac{5}{4}ev(y)\bar{\psi}\psi - \frac{e}{2}A'^M{}_M\bar{\psi}\psi + \frac{e}{2}A'_{MN}\bar{\psi}\gamma^{MN}\psi + \dots, \end{aligned} \quad (3.18)$$

and

$$\begin{aligned} & -\frac{1}{4}\text{Tr}(\mathcal{F}^{MN}\mathcal{F}_{MN}) \\ &= \frac{1}{16}ev^2(y)R - \frac{1}{2}\partial^4v(y)\partial_4v(y) - \frac{1}{16}v^2(y)C^P{}_{,MN}C^P{}_{,}{}^{MN} - \frac{1}{4}v(y)(\partial^4v(y))C^M{}_{,4M} \\ & \quad - \frac{5}{16}e^2v^4(y) - \frac{1}{16}e^2v^2(y)(6A'_{MN}A'^{MN} + 4A'^M{}_MA'^N{}_N - 2A'_{MN}A'^{NM}) \\ & \quad - \frac{1}{8}R^{PQ}{}_{,MN}R_{PQ}{}_{,}{}^{MN} - \frac{1}{4}F^{MN}F_{MN} + \frac{1}{4}eA'^M{}_PA'_Q{}^NR^{PQ}{}_{,MN} + \dots, \end{aligned} \quad (3.19)$$

where $C^P{}_{,MN}$ is the torsion:

$$C^P{}_{,MN} \equiv \Gamma^P{}_{MN} - \Gamma^P{}_{NM}. \quad (3.20)$$

Note that we obtain the Ricci curvature $R = b_a{}^Mb_b{}^NR^{ab}{}_{,MN}$.

When $v(y)$ is constant, if we choose a value of e such that

$$\frac{1}{16}ev^2 = \frac{1}{16\pi G}, \quad (3.21)$$

we obtain the Einstein-Hilbert term. From (3.18), the fermion mass m_F can be defined as

$$m_F^2 = \frac{25}{16}e^2v^2 = \frac{25}{2}eM_p^2, \quad (3.22)$$

where $M_p^2 \equiv \frac{1}{8\pi G}$ is the Planck mass. Since M_p^2 is very large, the coupling e must be very small to determine the fermion mass, which is in the order of GeV.

Let us assume that $v(y)$ takes the following form,

$$v(y) = v_0 \sin \frac{2\pi}{a} y, \quad (3.23)$$

where v_0 is a constant with a mass dimension of 1, and a is the radius of the fifth dimension. When the fifth dimension is curled into a very small circle, the mass of the zero-mode fermion is zero owing to $\int_0^a \sin \frac{2\pi}{a} y dy = 0$. In this case, the condition (3.22) is unnecessary.

Equation (3.19) has a quadratic term $R^{NC}{}_{,MN} R_{CD}{}^{,MN}$; this quadratic term is not observed in the Einstein theory. However, it is known that when the quantum effect is considered, higher order curvature tensors are observed in the effective Lagrangian.⁸⁾

The second method for obtaining R is based on conformal transformation. First, let us decompose A_{aM} as

$$A_{aM} = \frac{1}{2} \phi b_{aM} + A'_{aM}, \quad (3.24)$$

where ϕ is a real scalar field with a mass dimension of 1 and

$$A'_{aM} \equiv A_{aM} - \frac{1}{2} \phi b_{aM}. \quad (3.25)$$

By substituting (3.24) into (3.1) and using (3.17), we obtain

$$\begin{aligned} & -\bar{\psi} i \gamma^M \mathcal{D}_M \psi \\ & = -\bar{\psi} i \gamma^M \partial_M \psi - \frac{5}{4} e \phi \bar{\psi} \psi - \frac{e}{2} A'^M{}_M \bar{\psi} \psi + \frac{e}{2} A'^{MN}{}_{MN} \bar{\psi} \gamma^{MN} \psi + \dots, \end{aligned} \quad (3.26)$$

and

$$\begin{aligned} & -\frac{1}{4} \text{Tr}(\mathcal{F}^{MN} \mathcal{F}_{MN}) \\ & = \frac{1}{16} e \phi^2 R - \frac{1}{2} \partial^M \phi \partial_M \phi - \frac{1}{16} \phi^2 C^P{}_{,MN} C_{P,}{}^{MN} - \frac{1}{4} \phi (\partial^M \phi) C^N{}_{,MN} - \frac{5}{16} e^2 \phi^4 \\ & \quad - \frac{1}{8} R^{PQ}{}_{,MN} R_{PQ,}{}^{MN} - \frac{1}{4} F^{MN} F_{MN} + \frac{1}{4} e A'^M{}_P A'^N{}_Q R^{PQ}{}_{,MN} + \dots. \end{aligned} \quad (3.27)$$

The first term on the right-hand side of (3.27) is called nonminimal coupling. In other words, we obtain a Lagrangian of the scalar-tensor theory. We can show that a conformal transformation

$$g_{MN} \rightarrow g_{*MN} = \Omega^2(x) g_{MN} \quad (3.28)$$

gives

$$R = \Omega^2(x) (R_* + \dots), \quad \sqrt{-g} = \Omega^{-5}(x) \sqrt{-g_*}, \quad (3.29)$$

where $\Omega(x)$ is an arbitrary function of the space-time coordinate x . From Reference 9), if we choose $\Omega(x)$ such that

$$\frac{1}{16}e\phi^2\Omega^{-3}(x) = \frac{1}{16\pi G} \quad (3.30)$$

and introduce a scalar field $\sigma(x)$ that satisfies

$$\phi = \frac{2\sqrt{2}}{\sqrt{e}}M_p e^{\zeta\sigma} \quad \text{with} \quad \zeta^{-2} \equiv \left(\frac{16}{3} + \frac{8}{e}\right)M_p^2, \quad (3.31)$$

we can obtain

$$\sqrt{-g} \left(\frac{1}{16}e\phi^2 R - \frac{1}{2}\partial^M \phi \partial_M \phi \right) = \sqrt{-g_*} \left(\frac{1}{16\pi G} R_* - \frac{1}{2}\partial^M \sigma \partial_M \sigma \right), \quad (3.32)$$

and

$$\sqrt{-g} \frac{5}{4} e \phi \bar{\psi} \psi = \sqrt{-g_*} \frac{5\sqrt{2e}}{2} M_p e^{\zeta\sigma} \bar{\psi}_* \psi_*, \quad (3.33)$$

where we apply the transformation,

$$\psi \rightarrow \psi_* = \Omega^2(x)\psi, \quad \bar{\psi} \rightarrow \bar{\psi}_* = \Omega^2(x)\bar{\psi} \quad (3.34)$$

to ψ and $\bar{\psi}$. Note that the Einstein-Hilbert term and the mass term for the fermion are obtained simultaneously.

Reference 9) or 10) shows that the interaction term including the scalar field in Jordan frame can be eliminated from the Lagrangian in Einstein frame, if the Lagrangian possesses scale invariance in Jordan frame. The complete decoupling of the scalar field from the matter in Einstein frame does imply the weak equivalence principle (WEP), whereas (3.33) includes the coupling. Hence, our model might potentially violate WEP. This issue will remain a subject of future studies.

§4. Discussion

Here, we discuss the validity of the vacuum expectation value in (3.15). Equation (3.19) includes a quartic term of $v(y)$. However, for making an assumption that the local minimum of the potential is not zero, the quadratic terms of $v(y)$ must also be considered. The quadratic terms may be induced by radiative correction,^{11),12)} or the relevant $v^2(y)$ terms in (3.19),

$$\frac{1}{16}e v^2(y) R - \frac{1}{2}\partial^4 v(y) \partial_4 v(y) - \frac{1}{16}v^2(y) C^C{}_{,MN} C_C{}^{,MN} - \frac{1}{4}v(y)(\partial^4 v(y)) C^N{}_{,4N} \quad (4.1)$$

may induce the same effect as the quadratic terms.¹³⁾

For example, let us extend our model to six dimensions. If the extra space is a two-dimensional curved space of constant positive curvature a^{-2} , the six-dimensional Ricci curvature R can be written as

$$R = R_4 + \frac{1}{a^2}, \quad (4.2)$$

where R_4 represents the four-dimensional Ricci curvature. Then, we have the following quadratic term:

$$\begin{aligned} \mathcal{L}_6 &= \frac{1}{16}ev^2(y)R - \frac{5}{16}e^2v^4(y)\cdots \\ &= \frac{1}{16}ev^2(y)R_4 + \frac{1}{16a^2}ev^2(y) - \frac{5}{16}e^2v^4(y)\cdots \end{aligned} \quad (4.3)$$

Note that the relative sign of the $v^2(y)$ and $v^4(y)$ terms is negative. When a assumes an extremely small value, $v(y)$ takes an enormous vacuum expectation value. Hence, it is possible that A_{aM} takes the vacuum expectation value as (3.15).

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